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Multidimensional Poverty in Brazil in the Early 21st Century: Evidence from the Demographic Census

Adriana Stankiewicz Serra[1] · **Gaston Isaias Yalonetzky**[2] · **Alexandre Gori Maia**[1]

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Abstract

This paper examines multidimensional poverty in Brazil in 2000 and 2010, based on the microdata of the Demographic Censuses. Our analysis is disaggregated into five classes of municipalities according to their degree of urbanisation and remoteness, highlighting wide rural–urban inequalities in the levels and dynamics of poverty. We compare estimates of traditional monetary poverty with multidimensional poverty measures based on two methods: (i) the Alkire-Foster counting identification approach; and (ii) the Permanyer two-stage poverty identification approach. The two-stage approach introduces the concepts of complementarity/substitutability within and across poverty dimensions, which enables a more precise identification of the population targeted by anti-poverty policies. All methods highlight substantial progress in poverty alleviation. In absolute terms, the reduction in the incidence of multidimensional poverty was significantly larger in the initially poorest areas—rural and intermediate municipalities, as well as those in the North and North–East regions. Important advances were made in standard of living, especially in the access to electricity, durable consumer goods and private bathroom in the households in rural and intermediate municipalities. However, remote municipalities remain relatively poorer from any perspective, facing more difficulties in reducing monetary poverty.

Keywords Multidimensional poverty measurement · Robustness tests · Rural poverty · Urban poverty

JEL Classification I32 · R11

✉ Adriana Stankiewicz Serra
adri.stankiewicz@gmail.com

Gaston Isaias Yalonetzky
g.yalonetzky@leeds.ac.uk

Alexandre Gori Maia
gori@unicamp.br

1 Institute of Economics, University of Campinas (UNICAMP), Rua Pitágoras, 353, Barão Geraldo, Campinas, São Paulo 13083-857, Brazil

2 Leeds University Business School, Maurice Keyworth Building, Leeds LS6 1AN, UK

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1 Introduction

The first of the Sustainable Development Goals (SDGs) of the 2030 Agenda for Sustainable Development highlights the urgency to "end poverty in all its forms, everywhere" (UN 2019). Although reducing poverty in the income or consumption perspective remains the top priority in the developing world, other fundamental dimensions are also key to a decent life in modern societies, such as health, education, and living conditions. Essentially, poverty alleviation in its multiple dimensions implies overcoming capability deprivation (Sen 1999). We analyse the concepts and dynamics of multidimensional poverty in Brazil, where monetary poverty has markedly decreased over the past decades.

Despite overall improvements in socioeconomic indicators since the nineties, poverty alleviation remains a real challenge in Brazil. The World Bank (2020) indicates that the proportion of the population surviving on less than US$1.90 a day (at 2011 purchasing power parity—PPP) dropped from 21.6% in 1990 to 2.7% (5.5 million people) in 2014. Moreover, an economic recession in 2015–2016 reversed the decline of poverty, which reached 4.4% of the population in 2018 (9.2 million people). Based on the international poverty line of US$5.50 a day (2011 PPP), more suitable for upper-middle-income countries, 19.9% of the Brazilian population (41.7 million people) was poor in 2018 (World Bank 2020). The country also experienced meaningful progress in social indicators that are essential for human development, such as literacy and sanitation. The literacy rate of 15–24-year-olds increased from 94.2% in 2000 to 97.5% in 2010, while the proportion of households with poor sanitation[1] dropped from 14.0 to 8.1% (IBGE 2011a).

Brazil is also characterised by extreme regional inequalities, which reminds of the differences between developed and developing countries. Deprivations persist and remain substantially larger in rural areas and in the North and North–East regions of the country (IBGE 2011a). Considering a poverty line of a quarter of the minimum wage[2] (just under $2 a day in 2010), 39% of the rural population were poor in 2010, in contrast with 11.5% in the urban areas (IBGE 2011a). Differences between rural and urban areas are also remarkable in terms of other development indicators. One of the most concerning indicators is the illiteracy rate among people aged 15 and above, which ranged from 7.3% in urban areas to 23.2% in rural areas in 2010 (IBGE 2011b).

The empirical literature has extensively examined the determinants of the reduction in monetary poverty in Brazil. The main reasons for this progress were economic growth, labour market improvements, real minimum wage rises, expansion of social security and social assistance benefits, both in terms of coverage and value, and macroeconomic stability (Campello and Neri 2014; Ravallion 2011a; World Bank 2016a, b). Helfand and Del Grossi (2009) demonstrated that rural poverty decreased more rapidly than urban poverty between 1995 and 2006, mainly because of social security benefits. Helfand et al. (2009) showed that a fall in income inequality and an increase in cash transfer programs, such as *Bolsa Família* (family grant), contributed to reducing income inequality more remarkably

[1] Poor sanitation means that households lack access to any of the following services: general water supply, sanitary sewage or septic tank, and garbage collection (IBGE 2011a).

[2] Brazil does not have an official poverty line. The Brazilian government uses different income criteria to select beneficiaries of social programs. The *Benefício de Prestação Continuada* (BPC), for instance, which consists of a monthly allowance of a minimum wage, is aimed at the elderly and the disabled people with a monthly household income *per capita* below a quarter of the minimum wage. For more information on this and other social security benefits, see: https://www.inss.gov.br/beneficios/ (in Portuguese).

in rural than in urban areas between 1998 and 2005. Neri et al. (2012) observed that the share of labour income in the average household income in rural areas dropped from 81% in 1992 to 66.5% in 2009. The educational deprivation among the rural poor, including school attendance and adult education, is certainly one of the biggest challenges towards overcoming poverty, either through agricultural or non-agricultural activity (Balsadi 2012; Helfand and Pereira 2012).

In this study, we fill a gap in the literature on poverty in Brazil, where monetary poverty analysis is still predominant, and the rural/urban contrasts have not been examined deeply through a multidimensional lens. We propose a new indicator of multidimensional poverty in Brazil to address two main questions: (i) what has changed in other dimensions of poverty besides income?; and (ii) has poverty changed evenly within the country, i.e. what were the implications for rural–urban inequalities? We analyse the period 2000–2010, for two main reasons: (i) the availability of the individual-level data (microdata) from the two last Demographic Censuses, from the Brazilian Institute of Geography and Statistics (IBGE); and (ii) this is the recent period of most relevant socioeconomic changes in the country, which was particularly favoured by the world economic dynamics, and witnessed key social policy initiatives. Compared to the annual National Household Sample Survey (PNAD), namely the most frequently used data source for poverty studies in Brazil, the Demographic Census has some advantages. Its large sample size allows us to analyse the smallest administrative unit in the country, making it possible to compare the poverty dynamics between rural and urban municipalities. This is the first study to analyse poverty dynamics in the country at this level of detail.

The aim of this paper is then twofold. First, we estimate multidimensional poverty measures for Brazil in 2000 and 2010, comparing the results obtained from two methods: (i) the Alkire–Foster (AF) counting method (Alkire and Foster 2011; Alkire et al. 2015); and (ii) the two-stage identification approach suggested by Permanyer (2019). The comparison of these methods allows us to discuss the dynamics of multidimensional poverty using different concepts of poverty in Brazil, based on essential and policy-relevant well-being indicators beyond income.

Second, because of the significant regional inequalities in Brazil, we disaggregate the analyses into five classes of municipalities, according to the new rural–urban typology proposed by IBGE (2017b): predominantly urban, intermediate close to a city, intermediate remote, predominantly rural close to a city, and predominantly rural remote municipalities. In line with criteria widely used internationally, such as the OECD typology and European Union, this typology is mainly built on demographic density and distance to the most important urban centres. The differentiation between remote municipalities and those close to a city gives us more accuracy to evaluate how poverty has varied between less developed (rural and isolated municipalities) and more developed urban centres. We engage with the literature emphasizing the relevance of the relationships between rural areas and cities for rural development and poverty alleviation (Berdegué et al. 2012; Irwin et al. 2010; Schejtman and Berdegué 2004; Veiga 2003). The proximity to more urbanised regions with better infrastructure facilitates access to goods and services, and job opportunities, besides boosting both agricultural and non-agricultural activities. Therefore, in addition to assessing the associations in each year between (1) population density and poverty, and (2) distance to cities and poverty, we are interested in identifying whether there is any associative connection between these two municipal traits (density and distance) and *absolute poverty reduction*.

Our results show that poverty decreased remarkably in Brazil from 2000 to 2010, using all the aforementioned empirical strategies and in all classes of municipalities. Overall, the reduction in the incidence of poverty (i.e. the proportion of poor people) was relatively larger than that in the intensity of poverty (i.e. a measure of how far the poor are from the poverty line). The greatest falls in both monetary and non-monetary poverty occurred in intermediate and predominantly rural municipalities, which feature the highest proportion of poor people. Nevertheless, the incidence of poverty in intermediate and rural municipalities is still significantly higher than in urban municipalities, with the worst conditions among those living in remote municipalities. Income poverty remains the greatest deprivation in all classes of municipalities. There were substantial improvements in access to electricity, durable consumer goods and private bathrooms in the households in intermediate and rural municipalities. However, there are still considerable deprivations in sanitation and basic education among the population aged 15 and over. The cross-tabulations of monetary and non-monetary poverty headcounts also provided evidence of the importance of evaluating the joint distribution of deprivations. Only a proportion of the population deemed poor by either poverty perspective is simultaneously poor in both.

The paper has five sections. Following this introduction, Sect. 2 refers to the literature on multidimensional poverty measurement. Section 3 discusses the methodological choices for the empirical study and presents the data and the rural–urban typology for the Brazilian municipalities adopted in this investigation. Section 4 shows and examines the results. Finally, Sect. 5 makes the concluding remarks.

2 Multidimensional Poverty Measures

While there is a consensus regarding the multidimensionality of poverty, the same is not true about how to measure it. Poverty measurement involves two steps, as stated by Sen (1976): (i) the identification of the poor among the population; and (ii) the aggregation of the available information on the poor in a poverty index. In the monetary perspective, the identification of the poor is based on a poverty line, while the aggregation of information may result in different indices. The simplest and most commonly used measure is the headcount ratio—the proportion of people identified as poor in a population—which applies to both monetary and multidimensional poverty. However, by taking into account multiple dimensions, the measurement of poverty becomes more complex right at the identification step. First, a cut-off criterion or "deprivation line" must be defined for each dimension. Second, in order to identify the poor, we must decide how to aggregate different attributes.

The deprivation counting method has been extensively used to identify the poor at least since Townsend (1979). This identification method has two extreme approaches: the union and the intersection (Atkinson 2003). In the union approach, an individual is deemed multidimensionally poor if he/she is deprived in at least one dimension. In the intersection approach, an individual is poor only if he/she is deprived in all dimensions. Each of these criteria has the advantage of identifying the same individuals as multidimensionally poor, independently of how the deprivations are combined into a poverty identification score.

On the other hand, neither may be appropriate in terms of public policy. The union criterion tends to identify a large proportion of the population as poor. In contrast, the intersection criterion usually results in a tiny proportion of poor population. For these reasons, some intermediate criterion has been commonly used, such as in the global Multidimensional Poverty Index (MPI), developed by the Oxford Poverty and Human Development Initiative (OPHI) in partnership with the United Nations Development Programme (UNDP) (Alkire and Santos 2010; UNDP 2018). The global MPI identifies poverty calculating a weighted average of ten indicators, aggregated in three dimensions: (i) health (nutrition and child mortality); (ii) education (years of schooling and school attendance); (iii) standard of living (electricity, sanitation, drinking water, housing, cooking fuel, and assets). The weights are equally distributed among the dimensions, and equally divided among indicators within each dimension. A household (and everyone in it) is then considered multidimensionally poor if it is deprived in at least one-third of the weighted indicators (UNDP 2018).

The global MPI is an application of the Alkire-Foster (AF) method (Alkire and Foster 2011; Alkire et al. 2015). The World Bank (2018) used the same method to release a multidimensional poverty measure incorporating five dimensions of well-being: (i) monetary poverty; (ii) education; (iii) access to basic infrastructure; (iv) health and nutrition; (v) security.[3] The literature also provides several multidimensional poverty measurement methods, amid disagreements regarding the aggregation of different attributes into a scalar index. Ravallion (2011b, 2016) argues for a dashboard of disaggregated development indicators, such as the SDGs (UN 2019), while Alkire et al. (2015) advocate the synthesis of information on the multiple dimensions of poverty into a scalar index, such as the global MPI (Alkire and Santos 2010; UNDP 2018). Unlike the dashboard, a scalar index can be sensitive to the joint distribution of deprivations, i.e. the deprivations that each individual or household faces simultaneously. A scalar index is required to quantify the incidence and degree of concentration of multiple deprivations among the same individuals or in a particular dimension (Yalonetzky 2014), which is fundamental for social policy planning.[4] Moreover, an advantage of a scalar index is that it produces complete orderings of distributions in time and space (Ferreira and Lugo 2013)—for example, ranking of states or municipalities within the country. However, in the case of joint distribution, which requires a single source of data, the dimensions to build the index are limited to the variables investigated in the survey. But the main dimensions of poverty are not usually investigated in the same survey. For example, in Brazil, the main household surveys do not usually include any nutrition indicator, which is part of the health dimension in the global MPI.

Even though a scalar index may have the advantage of providing information on the joint distribution of deprivations, the arbitrariness in the selection of parameters necessary for aggregation in a single index is widely criticised (Ferreira and Lugo 2013; Ravallion 2011b, 2016). Hence the importance of implementing a robustness analysis comparing different parameters of aggregation (Alkire et al. 2015). For example, Ravallion (2011b) draws attention to the implicit trade-offs between the components of the MPI proposed by Alkire and Santos (2010). The choice of weights for the global MPI indicators implies that preventing the death of a child would be equivalent to eliminating deprivations in any three of six indicators in the standard of living dimension. Given that multidimensional analysis includes non-market goods, such as health and education attainments and access to public services, the author points out the lack of theoretical foundation for the weights structure

[3] The multidimensional poverty measure released for 119 countries contained only the first three dimensions. Six countries contained data for the five dimensions: Ecuador, Indonesia, Iraq, Mexico, Tanzania, and Uganda (World Bank 2018). It is worth noting that the Commission on Global Poverty did not recommend the inclusion of monetary poverty among the dimensions of the multidimensional poverty index (World Bank 2017).

[4] A scalar index is required for quantifying the incidence of multiple deprivations but not sufficient. For instance, the Human Development Index (HDI) is a scalar index that does not provide any information about the joint distribution of its dimensional indices—health, education, and income (UNDP 2018).

and the aggregation of indicators into a single measure. These decisions generally depend on the analyst and rarely reflect societal preferences.

The counting method makes it possible to analyse the overlap between deprivations, but its identification procedure may misclassify individuals or households into poor or non-poor (Permanyer 2019). The misclassification occurs because this method implicitly assumes perfect substitutability between indicators. Permanyer (2019) also suggests an alternative procedure to identify the poor, which requires a previous definition of substitutability or complementarity between indicators and between dimensions (poverty profiles).

Despite remarkable improvements in the measurement of multidimensional poverty, most studies in Brazil are still limited to monetary poverty, with a few exceptions. Bagolin and Ávila (2006) built a composite index from four capabilities or human needs indicators—essential nourishment, health, safety, and basic knowledge—to analyse multidimensional poverty among the Brazilian states. While this approach chose the best data source for each indicator, allowing the authors to incorporate important dimensions, it did not provide any information about the joint distribution of the deprivations. Barros et al. (2006) constructed a family poverty index including a total of 48 indicators in six dimensions: (i) vulnerability; (ii) access to knowledge; (iii) access to work; (iv) scarcity of resources; (v) child development; (vi) housing deprivation. Kageyama and Hoffmann (2006) combined income with other indicators to classify the poor into three groups: (i) income deprived; (ii) deprived in at least two of three basic facilities—piped water, bathroom and electricity; or (iii) deprived in income and all the other three indicators (extremely poor). While using four indicators only, this study offers a new perspective by combining the analysis of income deprivation with living conditions, through a counting approach (second group) and the intersection criterion (third group, deprived in all indicators). Buainain et al. (2013) examined rural poverty in Brazil through cross-tabulations of deprivations in income and other 16 indicators. Cobo et al. (2014) adapted the poverty measurement method of Mexico's CONEVAL (*Consejo Nacional de Evaluación de la Política de Desarrollo Social*) to the Brazilian Demographic Censuses 2000 and 2010. The authors presented municipal-level results for Brazil but did not address the differences between rural and urban areas.

We use the information available in the microdata of the Demographic Censuses 2000 and 2010 to propose new multidimensional poverty indices for Brazil. We compare two methods: the counting identification of the poor proposed by Alkire and Foster (2011) and the two-stage identification approach proposed by Permanyer (2019). The quality of the Census data also allows us to compare the dynamics of poverty accurately by groups of rural and urban municipalities.

3 Methods and Data

3.1 Poverty Measurement Methods

3.1.1 The Alkire-Foster Counting Method

The Alkire-Foster (AF) method (Alkire and Foster 2011; Alkire et al. 2015) consists of two steps: identification and aggregation. The identification step begins with the definition of the dimensions[5] to construct the multidimensional measure. The data must be available for every individual[6] to be able to examine the joint distribution of deprivations. The achievements of a population are represented by an $n \times d$ achievements matrix $\mathbf{X} = [x_{ij}]$, where $x_{ij} \geq 0$ is the achievement of individual i in dimension j for all $i = 1, \dots, n$ and for all $j = 1, \dots, d$. The achievements of any individual i in all d dimensions are represented by the d-dimensional vector $\mathbf{x}_i$ for all $i = 1, \dots, n$, which is row i of matrix $\mathbf{X}$. The achievements in any dimension j for all n individuals are represented by the n-dimensional vector $\mathbf{x}_j$ for all $j = 1, \dots, d$, which is column j of matrix $\mathbf{X}$.

The deprivation cut-off z_j defines the threshold below which an individual is deemed deprived for each dimension j. The set of deprivation cut-offs is represented by the d-dimensional vector $\mathbf{z} = [z_1, \dots, z_d]$. An individual i is considered deprived in dimension j if and only if $x_{ij} < z_j$. By applying the cut-offs $\mathbf{z}$ to the achievement matrix $\mathbf{X}$, one can obtain the deprivation matrix $\mathbf{g}^0 = \left[g_{ij}^0\right]$ such that $g_{ij}^0 = 1$ whenever $x_{ij} < z_j$ (deprived), and $g_{ij}^0 = 0$ otherwise (non-deprived). In other words, the matrix $\mathbf{g}^0$ represents the deprivations of all n individuals in all d dimensions. From the column vectors of the matrix $\mathbf{g}^0$, we calculate the proportion of the population deprived in each dimension, independently of other dimensions. The uncensored (raw) headcount ratio in dimension j is given by $h_j = (1/n)\sum_{i=1}^{n} g_{ij}^0$.

A multidimensional analysis based on a scalar index and the counting approach requires the choice of a weighting structure, which indicates the relative importance of deprivation in each dimension. The relative weight assigned to dimension j is denoted by w_j, such that $w_j > 0$ for all $j = 1, \dots, d$. The set of weights assigned to all d dimensions is given by the d-dimensional weighting vector $\mathbf{w} = [w_1, \dots, w_d]$. We adopted a normalised weighting structure (i.e. $\sum_{j=1}^{d} w_j = 1$).

From the weighting vector $\mathbf{w}$ and the deprivation matrix $\mathbf{g}^0$, we obtain the weighted deprivations matrix $\overline{\mathbf{g}}^0 = \left[\overline{g}_{ij}^0\right]$, such that $\overline{g}_{ij}^0 = w_j$ whenever individual i is deprived in dimension j, and $\overline{g}_{ij}^0 = 0$ otherwise. The sum of all entries in each row of matrix $\overline{\mathbf{g}}^0$, that is the weighted sum of deprivations in all d dimensions for an individual i, is the deprivation score c_i, such that $c_i = \sum_{j=1}^{d} \overline{g}_{ij}^0$, where $0 \leq c_i \leq 1$. The vector $\mathbf{c} = [c_1, \dots, c_n]$ denotes the deprivation scores for all n individuals.

In addition to the deprivation cut-offs, the AF method requires choosing a poverty cut-off k. An individual i is identified as multidimensionally poor if he/she is deprived in at least k weighted dimensions, with $0 < k \leq 1$. Formally, the identification function is given by $\rho_k(\mathbf{x}_i;\mathbf{z}) = 1$ if $c_i \geq k$ (poor), and $\rho_k(\mathbf{x}_i;\mathbf{z}) = 0$ otherwise (non-poor). By applying the poverty cut-off k and taking into account the focus on the population identified as multidimensionally poor, we obtain a new matrix from the deprivation matrix $\overline{\mathbf{g}}^0$, which is the censored deprivation matrix denoted by $\overline{\mathbf{g}}^0(k) = \left[\overline{g}_{ij}^0(k)\right]$. Formally, each element in $\overline{\mathbf{g}}^0(k)$ is given by $\overline{g}_{ij}^0(k) = \overline{g}_{ij}^0 \times \rho_k(\mathbf{x}_i;\mathbf{z})$ for all i and for all j. Thus, the rows containing the deprivation information of poor individuals remain unchanged, while those of non-poor individuals are censored, i.e. their deprivation status in every dimension become 0. The censored

[5] For simplicity of presentation, following Alkire et al. (2015), the term dimension here refers to each variable. In the empirical application, the term indicator refers to each variable, while dimension refers to groups of indicators.

[6] The term individual may refer to a person or a household, depending on the available data and the choice of the unit of identification.

deprivation score entry for individual i is given by $c_i(k) = \sum_{j=1}^{d} \bar{g}_{ij}^{0}(k) = c_i \times \rho_k(\mathbf{x}_i;\mathbf{z})$, such that $c_i(k) = c_i$ if $c_i \geq k$, and $c_i(k) = 0$ otherwise, where $0 \leq c_i(k) \leq 1$. The censored deprivation score vector for all n individuals is denoted by $\mathbf{c}(k) = \left[c_1(k), \ldots, c_n(k)\right]$.

Following the identification of the multidimensionally poor, we can define an aggregate measure. The focal measure in the AF method is the Adjusted Headcount Ratio ($M_0 = H \times A$): the product of the multidimensional headcount ratio (H), also called the incidence of multidimensional poverty, and the average intensity of multidimensional poverty (A). The incidence of poverty is simply the proportion of the population that is poor: $H = q/n$, where q is the number of individuals identified as poor using the dual cut-off approach ($\mathbf{z}$ and k). The intensity of poverty is the average deprivation score across the poor: $A = (1/q) \sum_{i=1}^{n} c_i(k)$. The M_0 measure can also be obtained as the mean of the vector of censored deprivation scores, which is the sum of the weighted deprivations that poor people experience, divided by the total population: $M_0 = (1/n) \sum_{i=1}^{n} c_i(k)$. The value of the three measures—H, A and M_0—may range between 0 and 1 (or 100%).

For any index Y, the absolute rate of change is the difference in values between two periods $t = 1$ (Y_1) and $t = 2$ (Y_2): $\Delta Y = Y_2 - Y_1$. The relative rate of change is the difference in the index as a percentage of its initial level and is $\Delta\%Y = \left(Y_2 - Y_1\right)/Y_1$. We can also decompose percentage changes in the adjusted headcount ratio with the formula: $\Delta\%M = \Delta\%H + \Delta\%A + \Delta\%H \times \Delta\%A$. The first term ($\Delta\%H$) is the percentage change in the number of multidimensionally poor people, the second term ($\Delta\%A$) is the percentage change in the average number of deprivations of the multidimensionally poor, and the final term ($\Delta\%H \times \Delta\%A$) is the multiplicative effect (Apablaza and Yalonetzky 2013).

As the AF method involves normative choices for indicators, deprivation cut-offs, weights, and poverty cut-off, any change in these parameters may affect the poverty estimates. In this regard, Lasso de La Vega (2010) provides dominance conditions to obtain unambiguous comparisons across two weighted deprivation score vectors regardless of the poverty cut-off and the poverty measure. The dominance conditions are implemented through dimension deprivation curves for the two most widely used poverty measures H and M_0. The first dominance condition is verified by representing H for all the admissible poverty cut-offs (k): if the curves of two weighted deprivation score vectors do not intersect, then all poverty measures satisfying the property of dimensional monotonicity[7] will agree in their ranking of the compared pair (even though H violates this principle). Similarly, the second dominance condition is confirmed if two weighted deprivation score vectors can be unanimously ranked by M_0 for all k. In this case, although M_0 is not distribution sensitive,[8] all poverty counting measures satisfying this property (in addition to dimensional monotonicity) will yield the same ranking. Even when the deprivation curves intersect, for both H and M_0, one can establish dominance conditions by restricting the set of poverty cut-offs, to guarantee the robustness of poverty comparisons in the counting approach. This is a useful tool in dominance analysis for changes in the poverty cut-off, particularly with ordinal and categorical variables.

[7] The dimensional monotonicity principle requires that if a poor person, who is not deprived in all dimensions, becomes deprived in an additional dimension then poverty should increase (Alkire et al. 2015). In other words, if the deprivation score of any individual identified as multidimensionally poor increases (decreases), then the overall poverty should also increase (decrease).

[8] Distribution sensitivity concerns the inequality among the poor. According to this principle, an increase (decrease) in overall poverty due to an increase (decrease) in the deprivation score of a multidimensionally poor person should be greater the higher his or her score is.

Besides checking the dominance conditions through the deprivation curves proposed by Lasso de La Vega (2010), we also tested the statistical significance of the changes in the poverty estimates over time, and the differences between the estimates of two classes of municipalities in the same year. If the confidence intervals of two estimates do not cross for any k, the differences are assumed to be statistically significant. Otherwise, it is essential to use an appropriate statistic to test whether the difference between two periods or groups of municipalities are statistically significant (Alkire et al. 2015). For each poverty cut-off, k, we applied the two-sample t-test to check the null hypothesis that the poverty measure (H or M_0, depending on the dominance condition) did not change between the two years (or between groups). We then used the intersection–union test (IUT) (Berger 1997), which rejects the null hypothesis against the alternative that the poverty measure decreased between the two years (or is lower in one of the groups) only if each of the individual hypotheses (i.e. each poverty cut-off, k) can be rejected. Each of the individual tests is performed at a significance level $\alpha = 1\%$, so that the overall test also has the same level α.

3.1.2 *The Two-Stage Identification* Approach

By comparing a poverty cut-off (k) against a weighted sum of deprivations, the counting method assumes that the indicators are perfect substitutes. Given the weighting structure of the global MPI, for instance, a deprivation in any of the health indicators is equivalent to a deprivation in any of the education indicators. Permanyer (2019) includes a hierarchical structure of variables (indicators) in mutually exclusive domains (dimensions) to address the issue of complementarity and substitutability between the components of the index. In this strategy, it is necessary to evaluate the possibility of the achievement in one or more indicators compensating for a deprivation in other(s), depending on how the indicators are grouped into different dimensions. The main idea is to define a combination of deprivations which represent the lack of a decent living condition in each dimension, rather than simply counting deprivations indistinctly.[9]

As mentioned in the previous section, the AF method's identification function satisfies the property of poverty consistency introduced by Lasso de la Vega (2010), which is defined in terms of the weighted deprivation score. Permanyer (2019) proposes a more general consistency condition, which can be applied to any identification function, based on two axioms: (i) non-triviality, meaning that the identification function is not constant across all deprivation profiles; and (ii) monotonicity, whereby whenever an individual i is identified as multidimensionally poor, another individual j experiencing deprivations at least in the same dimensions as those where i experiences deprivations, and possibly in others, should also be identified as poor. In other words, whenever an individual displays a combination of deprivations among d dimensions, defined by a set of poverty profiles (P_d), the individual is identified as poor.

Taking into account multiple dimensions, and given the partition of a set of variables across dimensions, the identification of the poor among the population in the hierarchical model suggested by Permanyer (2019) requires two definitions: (i) the deprivation status in each dimension (*within-dimension* identification function); and (ii) the overall deprivation across dimensions (*between-dimensions* identification function). Building the identification

[9] This section is based on Permanyer (2019) and also in a previous and extended version of it available at: https://www.ucm.es/data/cont/media/www/pag-37515/Permanyer_Mar16.pdf.

step on a set of axioms, the author demonstrates that independently of the weighting structure and the cut-offs, the counting method will not generate the same results of the two-stage identification functions.

In this approach, the identification step in the AF method can be modified from the deprivation matrix[10] $\mathbf{g}^0$, that provides the set or deprivation profiles in a given population, which is in turn obtained from the achievement matrix, $\mathbf{X}$. Each row of the matrix $\mathbf{g}_i^0 = \left[g_{i1}^0, \ldots, g_{id}^0\right]$ represents the deprivation profile of the individual i in all dimensions, where each element g_{ij}^0 is equal to 1 if individual i is deprived in dimension j, and 0 otherwise. Therefore, the profile $[0, \ldots, 0]$ represents an individual who is not deprived in any dimension, while the profile $[1, \ldots, 1]$ is deprived in all dimensions.

Formally, given a set of d variables in vector $\mathbf{D}$, these variables can be partitioned into S subsets of dimensions, $\left[\mathbf{D}_1, \ldots, \mathbf{D}_s\right]$, assuming that there are at least two dimensions, and that each group has at least two variables ($d_s := \left|\mathbf{D}_s\right| > 1$, with $d = \sum_s d_s$).[11] Then, it is necessary to evaluate the complementarity/substitutability within and between dimensions, in order to define the two identification functions, which may include any criterion of the counting approach, from union to intersection.

From the deprivation matrix $\mathbf{g}^0$ and the partition of the d variables into S dimensions, an individual is identified as deprived in dimension $\mathbf{D}_s$, with $s = 1, \ldots, S$, whenever he/she does not achieve minimum requirements for a decent living condition in that dimension. Adopting the counting approach to define the deprivation status in each dimension, an individual must be deprived in at least k_s^w attributes within dimension $\mathbf{D}_s$ in order to be deprived in that dimension, with $k_s^w \leq d_s$. The sum of the elements in each row of the deprivation matrix $\mathbf{g}^0$ in dimension $\mathbf{D}_s$, i.e. the deprivation of individual i in d_s variables, gives the number of deprivations c_{is}^w in that dimension, such that $c_{is}^w = \sum_{s=1}^{d_s} g_{is}^0$, with $0 \leq c_{is}^w \leq d_s$. Thus, the *within-dimension* identification function for each dimension $\mathbf{D}_s$ is given by $\rho_s^w\left(\mathbf{x}_i\right) = 1$ if $c_{is}^w \geq k_s^w$ (deprived), and $\rho_s^w\left(\mathbf{x}_i\right) = 0$ (non-deprived) otherwise. The deprivation status by dimension for all n individuals in all S dimensions is represented by a $n \times S$ matrix, $\mathbf{c}^w$, where each element is given by $\rho_s^w\left(\mathbf{x}_i\right)$. Therefore, the sum of the elements in each row of this matrix provides the number of dimensions in which an individual i is deprived, such that $c_i^b = \sum_{s=1}^{S} \rho_s^w\left(\mathbf{x}_i\right)$, with $0 \leq c_i^b \leq S$. Finally, an individual is identified as multidimensionally poor whenever he/she is deprived in at least k^b dimensions ($k^b \leq S$). Considering the deprivation matrix by dimension $\mathbf{c}^w$, the *between-dimensions* identification function is given by $\rho^b\left(\mathbf{x}_i\right) = 1$ if $c_i^b \geq k^b$ (poor), and $\rho^b\left(\mathbf{x}_i\right) = 0$ (non-poor) otherwise.[12]

With the definition of a threshold within each dimension (k_s^w) and a threshold between dimensions (k^b), Permanyer (2019) generalises the traditional counting method to the multiple dimensions' context. This is a fundamental difference regarding the AF method, that applies a single poverty cut-off (k) across all dimensions to identify the multidimensionally poor, besides a weighting distribution ($\mathbf{w}$) across dimensions. As demonstrated by the author, there is no combination of parameters k and $\mathbf{w}$ such that the identification function in the AF method coincides with the two-stage identification functions of the two-stage identification approach.

[10] In the two-stage identification approach, it is worth noting that the weighting structure of the AF method is not used, as the identification of the poor is based on the deprivations matrix $\mathbf{g}^0$, and not the weighted matrix $\bar{\mathbf{g}}^0$.

[11] Some notations here differ from those presented by Permanyer (2019), in order to keep the same notation of the AF method in Sect. 3.1.1, and prevent any ambiguous interpretation.

[12] The superscripts w and b refers to *within-dimension* and *between-dimensions*, respectively.

3.2 Data

We use the microdata sample files of the Demographic Census in 2000 and 2010 (IBGE 2017a), which are representative across the Brazilian municipalities. The Censuses use a stratified sampling procedure, where the strata are the Census tracts[13] and the Primary Sample Units (PSUs) are the households, which are sampled independently within each stratum. The investigation is carried out for every resident in the households selected for the sample. We take into account only permanent private households, excluding roomers, domestic workers, and relatives of domestic workers. The reason is that the IBGE excludes residents under such conditions in the calculation of the household income, and some information used to build the multidimensional poverty measures is collected only from permanent private households. Therefore, the resulting samples contain 20,029,532 individuals in 2000 (98.8% of the original sample, whose sampling rate was 11.7%), and 20,498,310 individuals in 2010 (99.3% of the original sample, sampling rate of 11.0%).

Based on the typology proposed by IBGE (2017b), we classified municipalities in five categories: (i) predominantly urban; (ii) intermediate close to a city; (iii) intermediate remote; (iv) predominantly rural close to a city; and (v) predominantly rural remote municipalities. This classification considers three main criteria: population density, population size, and location. The first criterion, population density, is in line with internationally adopted typologies, as those of the OECD and the European Union. From the IBGE statistical grid (1 km by 1 km cells), a cell must have a population density above 300 inhabitants per square kilometre, *and* the sum with its eight adjacent cells must have a population larger than or equal to 3000 inhabitants to be classified as a dense occupation cell. If these two conditions are not met, the cell is classified as a sparse occupation. Thus, each population unit (a municipality or cluster of municipalities[14]) is classified into three categories, according to the population size living in dense occupation areas *and* the proportion of the population in dense occupation areas to the total population: (i) predominantly urban municipality; (ii) intermediate municipality; or (iii) predominantly rural municipality. Table 1 presents the classification based on the combination of these two criteria.

The third criterion, location, is defined in terms of the commuting time to the nearest major urban centres.[15] An intermediate or rural municipality is classified as adjacent to a city if its relative distance to at least one important urban centre, as defined by IBGE, is equal or under the national mean. Otherwise, the municipality is classified as remote. Table 2 shows the results of this classification in terms of municipalities and population size for 2010. These criteria estimate that 76% of the total population lived in predominantly urban municipalities, instead of 84% from the official definition (IBGE 2011b, 2017b).[16] Figure 1 presents the map of Brazilian municipalities classified into five categories of the rural–urban typology.

[13] The Census tract is the minimum territorial unit—subdivisions of a municipality or municipality equivalent—for data collection.

[14] Two or more municipalities with strong population integration due to commuting to work or study, or contiguity between urban areas (in Portuguese, *arranjos populacionais*).

[15] Such as established by IBGE in the REGIC's project (*Regiões de Influência das Cidades*), the three higher levels in the hierarchy: metropolis, regional capital, and sub-regional centre.

[16] As 58 new municipalities were created between 2000 and 2010, the same rural–urban typology of 2010 is assumed in the year 2000.

Table 1 Conceptual matrix for the municipal rural–urban typology—Brazil—2010. *Source*: IBGE (2017b)

Ranges of the population in areas of dense occupation	Percentage distribution of the population in areas of dense occupation			
	More than 75%	From 50 and 75%	From 25 to 50%	Less than 25%
More than 50,000 inhabitants	Predominantly urban	Predominantly urban	Predominantly urban	Predominantly urban
From 25,000 to 50,000 inhabitants	Predominantly urban	Predominantly urban	Intermediate	Predominantly rural
From 10,000 to 25,000 inhabitants	Predominantly urban	Intermediate	Predominantly rural	Predominantly rural
From 3,000 to 10,000 inhabitants	Intermediate	Predominantly rural	Predominantly rural	Predominantly rural
Less than 3,000 inhabitants	Predominantly rural	Predominantly rural	Predominantly rural	Predominantly rural

Table 2 Municipalities and population, according to the rural–urban typology—Brazil—2010. *Source*: IBGE (2017b)

Rural–urban typology	Municipalities		Population	
	Number	%	People	%
Predominantly urban	1,456	26.16	144,765,500	75.89
Intermediate close to a city	686	12.33	12,461,908	6.53
Intermediate remote	60	1.08	1,305,906	0.68
Predominantly rural close to a city	3,040	54.63	28,697,888	15.04
Predominantly rural remote	323	5.80	3,524,597	1.85
Total	5,565	100.00	190,755,799	100.00

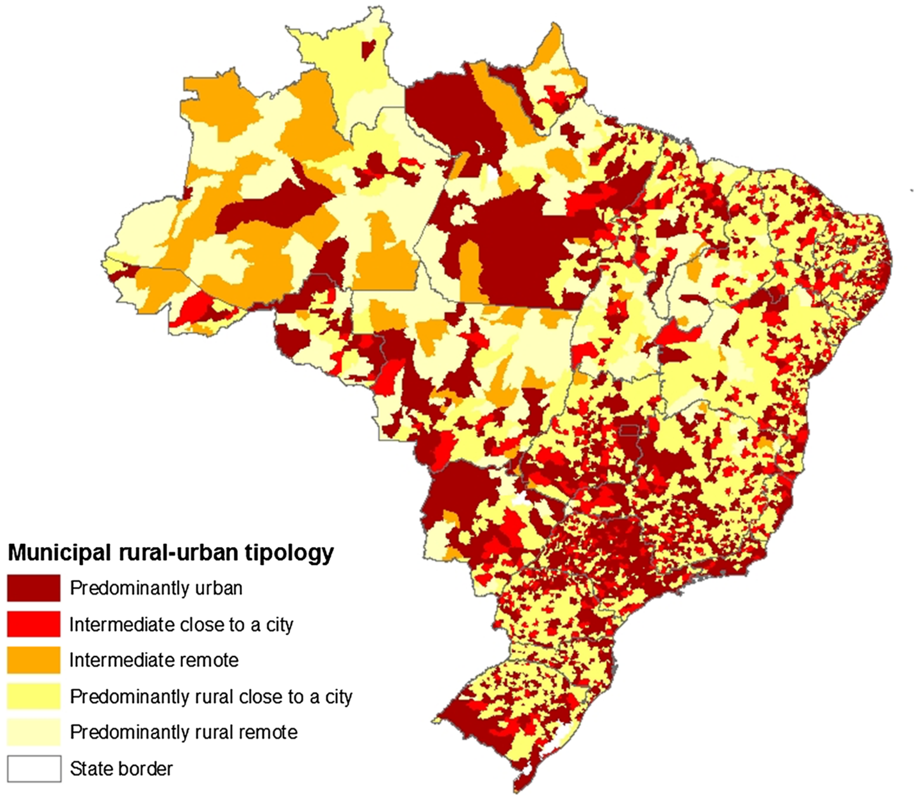

Fig. 1 Municipal rural–urban typology—Brazil—2010. *Data source*: IBGE (2017b)

The spatial distribution of municipalities according to the rural–urban typology across the Brazilian territory (Fig. 1) shows that the great majority of predominantly urban municipalities are located in the most developed areas of the Centre-South region and in the North–East coast. The intermediate and rural municipalities are evenly distributed among the national territory. Meanwhile, the municipalities classified as remote are concentrated in the Amazon region and in the *Matopiba* region (part of Maranhão, Tocantins, Piauí, and Bahia States), which is the most recent agricultural frontier in the country.

3.3 Multidimensional Poverty Measures for Brazil

A multidimensional poverty measure requires some normative choices, as described by Alkire et al. (2015). In this study, the purpose of the measure is to compare the poverty dynamics across groups of rural and urban Brazilian municipalities between 2000 and 2010. The poverty measurement is based on the capabilities approach of Sen (1999), according to which poverty is the deprivation of human freedom, i.e. the lack of basic functionings ("doings and beings") that a person has reason to value. In this perspective, minimally adequate living conditions and basic education are essential requirements to function in society, and therefore compose the multidimensional poverty measure proposed for Brazil.[17]

Ideally, the national poverty measure should include at least the same three dimensions of the global MPI—health, education, and standard of living (UN 2019).[18] However, we were not able to compute the indicators in the health dimension for two reasons. First, the Demographic Censuses does not provide any indicator of nutrition. Second, although the Census provided information on child mortality in 2010 (IBGE 2017a), the same variables were not investigated in 2000. Implicitly among other things, this data limitation means that the poverty measures that we can construct with the available data will be placing zero weight on the missing indicators. Concomitantly, it will be harder to identify those most multiply deprived among the poor. On the other hand, we did include sanitation indicators in the standard of living dimension. Sanitation indicators, such as access to drinking water and sewage infrastructure, are often strongly correlated to health indicators, such as child morbidity and mortality from diarrhoea (Heller 1997). This study also points out the health risks arising from the lack of garbage collection or inadequate waste disposal, either by direct or indirect contact (through air, water, or soil) or through vectors (mosquitoes, cockroaches, rodents, among others). Clearly, securing access to sanitation services is not sufficient for good health. Hygienic practices, such as hand washing, house cleaning, and care in food preparation, are necessary conditions for a healthy life. In this sense, education and access to information play a vital role.

Our unit of poverty identification and analysis is the individual. This choice is also grounded on the ethical premise that education is an individual human right. Therefore it would not suffice having any household member with a minimum schooling achievement to identify everyone in that household as non-deprived in education, as is the case of the deprivation cut-off defined by the global MPI (UNDP 2018).

Table 3 describes the dimensions and indicators of our multidimensional poverty index (MPI). We chose the indicators based on the dimensions of poverty and welfare pointed out in the literature (Campello et al. 2014; Narayan et al. 2000; Stiglitz et al. 2009; UN

[17] In the study *Consultations with the Poor* in Brazil (World Bank 1999), which was part of a global research in 23 countries using participatory methods (Narayan et al. 2000), the adequate provision of basic public services (sanitation, education, infrastructure, and health services) was deemed a precondition to overcome poverty.

[18] The literature offers alternative normative criteria for the selection of dimensions and respective indicators. For instance, in contrast to participatory methods (e.g. Narayan et al. 2000) yielding a context-specific response, Nussbaum (2003, 2011) deduces a list of core capabilities meant to have universal validity.

Table 3 MPI-Brazil: dimensions, indicators, deprivation cut-offs and weights

Dimension and indicator	Deprivation cut-off	Weight (%)
Standard of living		50.00
Water pipes	The household has no plumbing system	8.33
Private bathroom	The household has no private bathroom	8.33
Waste disposal	No household garbage collection service in urban areas. No garbage collection from central refuse containers in urban and rural areas	8.33
Electricity	The household has no electricity	8.33
Durable consumer goods	The household: (i) does not have a refrigerator; or (ii) does not own at least one of the following: radio, TV, or telephone[a]; and (iii) does not own any of the following: washing machine, computer or car	8.33
Overcrowding	The household has more than two people per room	8.33
Education[b]		50.00
School attendance and literacy	7–17 years: the child or adolescent is not attending school	25.00
	18 years and above: the person cannot read and write	
Schooling gap and schooling achievement	7–8 years: the child is not attending school	25.00
	9–17 years: (i) schooling gap of two or more years; or (ii) the child or adolescent cannot read and write	
	18–64 years: (i) the person has not completed eight years of schooling[c]; or (ii) the person cannot read and write	
	65 years and older: the person cannot read and write	

[a]Landline or mobile phone. Mobile phone only investigated in the Demographic Census 2010

[b]Children from 0 to 6 years old cannot be classified because prior to 2010 education was not compulsory for children under 7 years old

[c]According to the Brazilian legislation on education prior to 2010 (Law no. 5692/1971). Currently, the Federal Constitution establishes as a duty of the State the guarantee of compulsory and free basic education for the 4–17 age group (Brasil 2016)

2019; UNDP 2018; World Bank 1999), the Brazilian legislation (Brasil 2014, 2016), and the constraints imposed by the variables investigated in the Demographic Censuses. As in the global MPI (UNDP 2018), the weights are equally distributed among the dimensions and equally divided among indicators within each dimension.

In the case of the AF method, we estimated the multidimensional poverty measures for all possible values of poverty cut-offs (k), combining the deprivation indicators and their respective weights. We performed a robustness analysis of the multidimensional poverty in Brazil, comparing the results between the two years—2000 and 2010—and across the municipal rural–urban typology.

In the case of the two-stage identification approach proposed by Permanyer (2019), we defined the within-dimension identification function using the intermediate counting approach for the standard of living and the intersection approach for education. Regarding the first dimension, we assume that a person is deprived in standard of living if he/she is deprived in at least half of the indicators, i.e. deprived in at least three of the six indicators. In this way, we consider that the existence of water pipes, private bathrooms, electricity, refrigerator, and at least one channel of communication or access to information (radio, TV or telephone) are minimum conditions to enjoy a decent life. Concerning the second dimension, a person must be deprived in all indicators to be deprived in education, with different criteria by age group. A person is fully deprived in education if he/she is: (i) 7–8 years old and is not attending school; (ii) 9–17 years old and is not attending school and is illiterate; (iii) 18 years old or more and is illiterate. The between-dimensions identification function is based on the union approach, i.e. an individual is identified as multidimensionally poor if he/she is deprived in any of the two dimensions. In other words, standard of living and education are complements in this approach.

We also compared the measures of monetary and non-monetary poverty. We assume that a person is deprived in income if the monthly household income *per capita* is up to R$ 140 in 2010 constant values, which was the poverty line for the beneficiaries of the *Bolsa Família Program* in July 2010.[19]

The first step before calculating any multidimensional poverty index is to check the joint distribution of the components to examine their level of relationships and identify redundancies. For example, Table 4 shows that 17.8% of the population lived in a household deprived in income, and 8.3% were deprived in school attendance and literacy. Nevertheless, only 2.4% of the population experienced both deprivations simultaneously. Overall, the low simultaneity among deprivations indicates the importance of multidimensional poverty analysis. In other words, we cannot infer deprivation in any indicator from a deprivation in another—monetary or non-monetary—indicator.

From cross-tabulations of deprivation in pairwise indicators, we can obtain two measures that help to understand the relationships between them: the Cramer's V correlation coefficient and the measure of redundancy or overlap R^0 (Table 5). The Cramer's V coefficient is the product of the matches (the percentage of people simultaneously deprived or not deprived in two indicators) minus the product of the mismatches (the percentage of people deprived in only one of two indicators), divided by the squared root of the product of the marginal distributions (the percentage of people deprived or not deprived in each indicator). The R^0 measures the percentage of people simultaneously deprived in two indicators, as a proportion of the minimum of the two uncensored or censored headcount ratios

[19] It was roughly equivalent to the international poverty line of US$ 2 per person per day (2011 PPP) (Campello and Neri 2014).

Table 4 Deprivation in pairwise indicators—Brazil—2010. *Data source*: Demographic Census (IBGE 2017a)

	Household income *per capita*	Water pipes	Private bath-room	Waste disposal	Electricity	Durable consumer goods	Overcrowding	School attend-ance and literacy	Schooling gap and schooling achieve-ment
Household income *per capita*	17.8								
Water pipes	3.8	6.8							
Private bathroom	4.4	3.9	7.4						
Waste disposal	7.8	5.2	5.6	20.5					
Electricity	0.9	1.1	1.0	1.3	1.4				
Durable consumer goods	3.6	2.6	3.0	3.8	1.3	6.0			
Overcrowding	9.4	3.4	4.1	7.5	0.8	3.1	27.8		
School attendance and literacy	2.4	1.5	1.7	3.4	0.4	1.4	2.6	8.3	
Schooling gap and schooling achievement	7.8	3.6	4.0	9.4	0.9	3.3	9.9	7.2	31.4

Table 5 Correlation and redundancy measures—Brazil—2010. *Data source*: Demographic Census (IBGE 2017a)

Dimension and indicator	Cramer's V	Redundancy (R^0)
Standard of living		
Water pipes & private bathroom	0.519	0.577
Water pipes & waste disposal	0.379	0.772
Water pipes & electricity	0.331	0.761
Water pipes & durable consumer goods	0.362	0.428
Water pipes & overcrowding	0.134	0.501
Private bathroom & waste disposal	0.389	0.760
Private bathroom & electricity	0.303	0.732
Private bathroom & durable consumer goods	0.407	0.495
Private bathroom & overcrowding	0.171	0.550
Waste disposal & electricity	0.209	0.908
Waste disposal & durable consumer goods	0.272	0.639
Waste disposal & overcrowding	0.100	0.367
Electricity & durable consumer goods	0.422	0.894
Electricity & overcrowding	0.078	0.570
Durable consumer goods & overcrowding	0.133	0.515
Education		
School attendance and literacy & schooling gap and schooling achievement	0.360	0.868
Income &		
Water pipes	0.268	0.558
Private bathroom	0.311	0.598
Waste disposal	0.271	0.440
Electricity	0.150	0.655
Durable consumer goods	0.278	0.599
Overcrowding	0.257	0.526
School attendance and literacy	0.088	0.289
Schooling gap and schooling achievement	0.127	0.440

(Alkire et al. 2015).[20] While the correlations between the pairs of indicators (Cramer's V) are not high, some pairs featured high redundancy (R^0). The highest redundancy occurs between waste disposal and electricity: 90.8% of the people who are deprived in the indicator having the lower marginal headcount ratio (1.4% of the population has no access to electricity) are also deprived in the other indicator (20.5% of the population with no household garbage collection). The near-universal access to electricity in Brazil may explain this result, as well as the likewise high value of R^0 between electricity and durable consumer goods (89.4%). Clearly, the use of durable goods such as refrigerators or TVs in the household depends on the existence of electricity. However, the opposite is not true, as a household may have electricity and may not have access to durable goods. This situation is more

[20] Regarding cross-tabulations and equations for computing the measures of correlation (Cramer's V) and redundancy (R^0), see Alkire et al. (2015, Sect. 7.3).

usual in rural areas. According to the Demographic Census 2010 (IBGE 2017a), 93% of the rural population (official definition in Brazil) had access to electricity, while only 79% had a refrigerator in the household.

The redundancy is also high between the indicators of education, which is justified by how the indicators are computed. For children from 7 to 8 years of age, the deprivation in the first indicator implies deprivation in the second one. The same justification applies to illiterate adults (18 years and above), which means full deprivation in education. Finally, it is important to notice the absence of high correlation or redundancy between the household income and the MPI components. The highest value of R^0 is for the income-electricity pair (65.5%). Therefore, the importance of including non-monetary measures in the poverty estimates is rendered manifest.

Table 6 shows the distribution of the population deprived in each indicator regardless of their poverty status (uncensored headcount ratios) in 2000 and 2010. The changes between 2000 and 2010 are statistically significant at 1% for all indicators. Despite the improvements in living conditions over time, deprivations remain remarkably high, particularly in relation to basic sanitation. The lowest headcount ratio (1.4%) is for the access to electricity, representing nearly 2.7 million people in 2010. Considering the education variables, although the deprivation cut-offs are the same for the age group up to 17 years, we broke down the results according to the Brazilian education system—elementary education (*Ensino Fundamental*) for the 7–14 age group (according to the Brazilian legislation on education before 2010) and secondary education (*Ensino Médio*) for the 15–17 age group. And again, in spite of the improvements in all indicators, much remains to be done to expand education opportunities, especially concerning the school attendance of the 15–17 age group, the schooling gap of two or more years for students between 9 and 17 years, the low schooling achievement and illiteracy among the adult population. For example, in 2010, 10% of the population aged 18 years and above was not able to read or write, which is dismal given the existing information and communication technologies in the twenty-first century.

Figure 2 shows a larger drop in deprivations in the most vulnerable (non-urban) areas, although people not living in the urban municipalities are still the most deprived in all indicators. The situation is worse among those living in remote areas. In 2010, more than a quarter of the population in remote municipalities lived in households with no plumbing system, and more than a third had no private bathroom. People living in intermediate and predominantly rural municipalities also experience high deprivations in education. For instance, while almost 7% of the urban residents were illiterate in 2010, this deprivation reached 19% in intermediate and 22% in rural municipalities (IBGE 2017a, b).

One main change between 2000 and 2010 was the expansion of access to electricity in rural areas. This result may be related to public policies such as *Luz para Todos* (Light for All), the rural electrification program established by the Federal Government in 2003. The program had benefited 13.3 million people by the end of 2010 (IICA 2011).[21]

[21] By the end of 2019, the *Luz para Todos* program had reached 16.8 million people in the Brazilian countryside. Program results, including data by state and region, are available at: https://eletrobras.com/pt/Paginas/Luz-para-Todos.aspx.

Table 6 Uncensored headcount ratio (h)—Brazil—2000 and 2010

Dimension and indicator	Uncensored headcount ratio			
	2000 (%)	2010 (%)	Absolute change (p.p.)	Relative change (%)
Standard of living				
Water pipes	12.3	6.8	− 5.5	− 44.9
Private bathroom	18.9	7.4	− 11.5	− 61.0
Waste disposal	28.2	20.4	− 7.7	− 27.4
Electricity	6.5	1.4	− 5.1	− 78.3
Durable consumer goods	17.6	6.0	− 11.6	− 66.0
Overcrowding	39.1	27.8	− 11.3	− 28.9
Education				
School attendance and literacy	11.0	8.3	− 2.7	− 24.5
7–17 years	10.1	6.9	− 3.2	− 31.6
7–14 years	5.4	3.1	− 2.3	− 42.7
15–17 years	22.1	16.6	− 5.4	− 24.7
18 years and above	13.7	10.0	− 3.8	− 27.5
Schooling gap and schooling achievement	41.6	31.4	− 10.3	− 24.7
7–8 years	6.3	2.5	− 3.8	− 60.3
9–17 years	34.2	20.2	− 14.0	− 41.0
9–14 years	30.6	17.3	− 13.3	− 43.6
15–17 years	41.0	25.8	− 15.2	− 37.0
18–64 years	56.6	41.1	− 15.5	− 27.4
65 years and above	37.0	29.0	− 7.9	− 21.5

Data source: Demographic Census (IBGE 2017a)

All changes are statistically significant at $\alpha = 1\%$. Due to rounding, there may be some minor discrepancies in reported numbers

4 Results

4.1 Counting Identification of the Poor

The measurement of multidimensional poverty following the rural–urban typology of municipalities provides essential insights into the territorial distribution of poverty in Brazil. First, for most values of the poverty cut-off (k), there is a clear distinction between the predominantly urban municipalities and the group of intermediate and predominantly rural municipalities for all three measures—H (Fig. 3), A (Fig. 4), and M_0 (Fig. 5). These differences narrowed from 2000 to 2010, but poverty remains relatively higher in rural and intermediate municipalities (the statistical significance of these differences is discussed in Sect. 4.2). Disparities are also more pronounced in the indicator of incidence than in the indicator of intensity of poverty.

The second insight comes from the poverty rankings of the classes of rural and intermediate municipalities, taking into account their location in relation to relevant urban centres. The ranking, in descending order of poverty, is as follows: rural remote, intermediate remote, rural close to a city, and intermediate close to a city. For all values of k, both

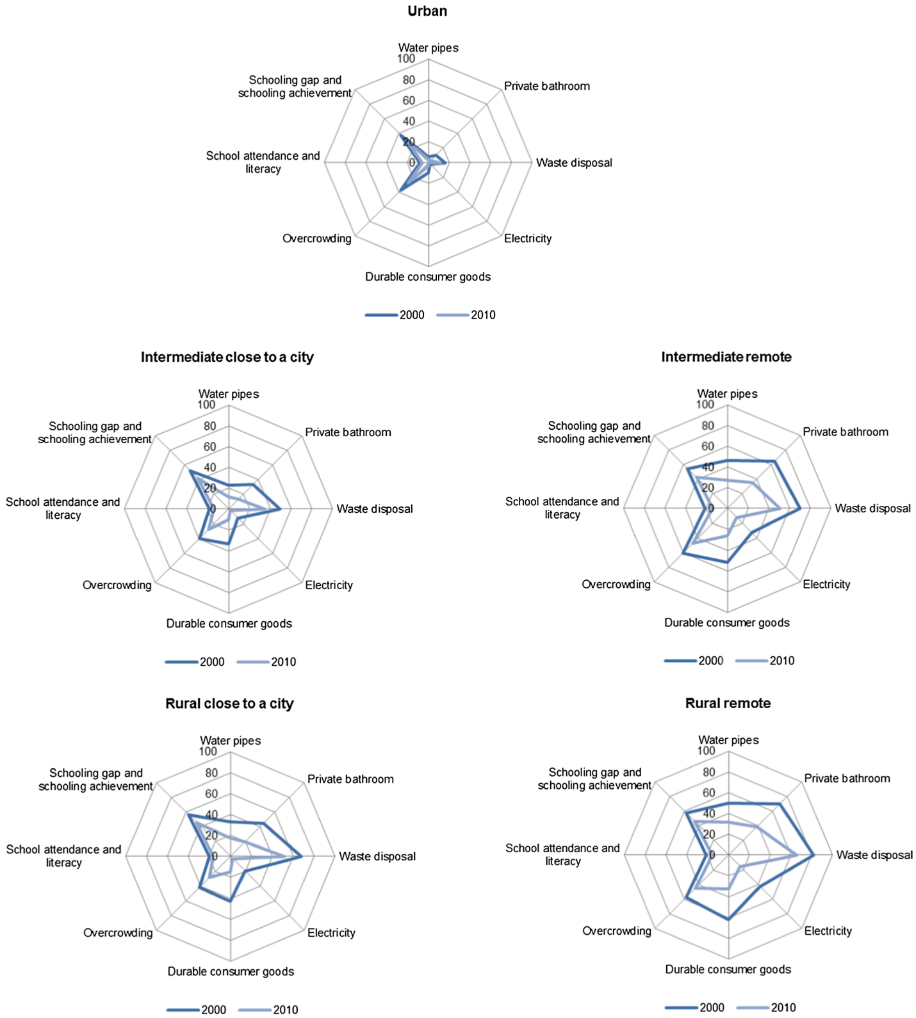

Fig. 2 Uncensored headcount ratio (h%), by class of municipality—Brazil—2000 and 2010. *Data source*: Demographic Census (IBGE 2017a)

multidimensional headcount ratio (H) and the adjusted headcount ratio (M_0) are higher in intermediate remote municipalities than in rural municipalities close to a city. In other words, independently of the classification into intermediate/rural, remote municipalities are poorer than those near to cities. This result evidences the importance of location for rural development (Berdegué et al. 2012; Irwin et al. 2010; Schejtman and Berdegué 2004; Veiga 2003), which overtakes the population size and density criteria in our poverty analysis for Brazil.

Figure 6 shows the decomposition of the changes in Adjusted Headcount Ratio (M_0) between 2000 and 2010 according to the rural–urban typology and poverty cut-offs (k), based on the method proposed by Apablaza and Yalonetzky (2013) (see Sect. 3.1.1). Except for the union criterion ($k = 8.33\%$) in intermediate and predominantly rural municipalities, the key driver of changes in M_0 was the change in the incidence of poverty (ΔH).

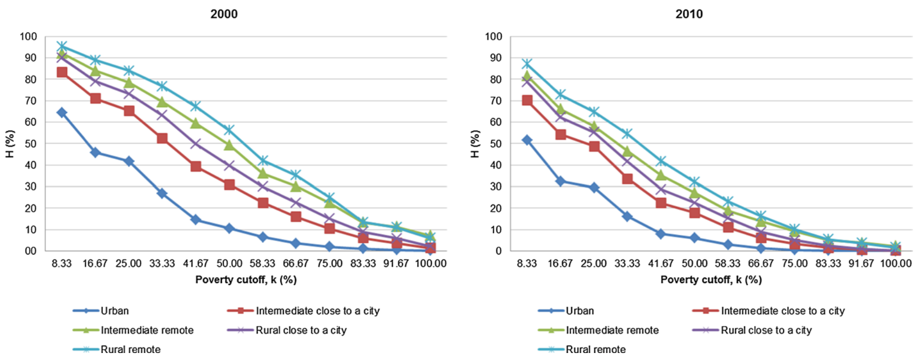

Fig. 3 Multidimensional headcount ratio ($H\%$), by class of municipality—Brazil—2000 and 2010. *Data source*: Demographic Census (IBGE 2017a)

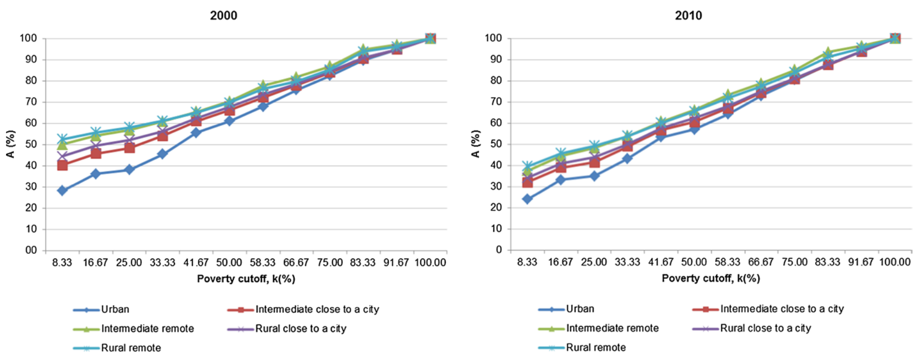

Fig. 4 Average deprivation share among the poor ($A\%$), by class of municipality—Brazil—2000 and 2010. *Data source*: Demographic Census (IBGE 2017a)

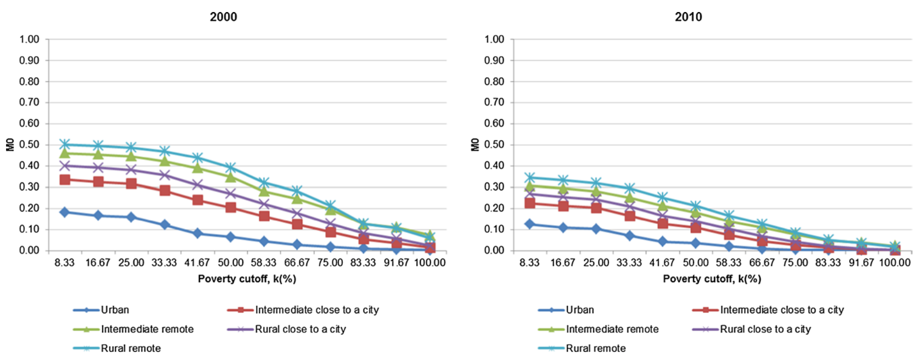

Fig. 5 Adjusted headcount ratio (M_0), by class of municipality—Brazil—2000 and 2010. *Data source*: Demographic Census (IBGE 2017a)

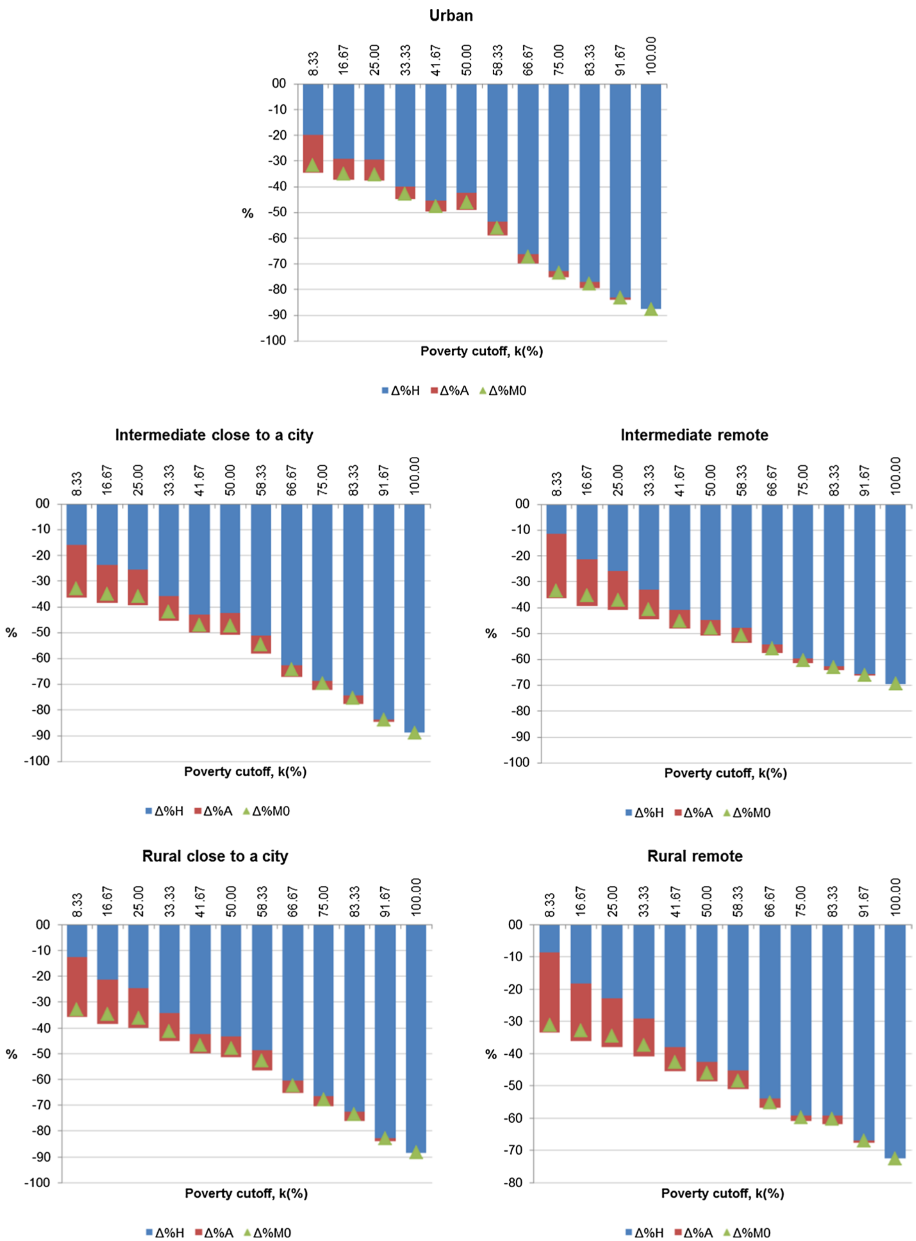

Fig. 6 Decomposition of the adjusted headcount ratio (M_0), by class of municipality—Brazil—2000 and 2010. *Data source*: Demographic Census (IBGE 2017a)

4.2 Robustness Analysis

In order to check the robustness of our results to alternative poverty cut-offs (k) and poverty measures, we used deprivation curves to verify the dominance conditions of Lasso de La Vega (2010). Figure 3 represents the first dominance condition concerning the multidimensional headcount ratio (H).[22] When the curves of H for all the admissible values of k do not intersect, all poverty measures satisfying the dimensional monotonicity property will lead to the same conclusion. Figure 5 illustrates the second dominance condition regarding the adjusted headcount ratio (M_0), which is confirmed when the curves of M_0 can be unanimously ranked for all k. Consequently, all poverty measures that fulfil monotonicity and distribution sensitivity properties will provide the same ranking.[23]

Some differences between the classes of municipalities in Figs. 3 and 5 are not visually apparent. However, the statistical analysis demonstrates that the dominance conditions hold for all $k \leq 0.75$ in 2000 and 2010. The ranking in descending order, according to both H and M_0, is: rural remote, intermediate remote, rural close to a city, intermediate close to a city, and urban municipalities. Most differences are statistically significant at 1%, including those differences between two years for each class of municipality.[24] The few exceptions are some of the differences between rural remote and intermediate remote municipalities in each year. For $k = 83.33\%$, the differences for both H and M_0 are only statistically significant in 2010. For $k \geq 91.67\%$, poverty becomes higher in intermediate remote than in rural remote municipalities, but when $k = 91.67\%$ the difference is statistically significant at 1% only for M_0 in 2010.

This robustness analysis provides two main findings. First, independently of the poverty cut-off, poverty is higher in remote municipalities when compared to those close to a city, whether they are classified as intermediate or predominantly rural municipalities. Second, the proof of dominance conditions for $k \leq 0.75$ is substantial when examining poverty across the municipal rural–urban typology and over time. Values of k that are higher than 0.75 are not usually chosen in counting approach implementations, as they tend to identify a tiny percentage of the population as poor.

4.3 Two-Stage Identification of the Poor

Figure 7 shows the results for all three measures—H, A, and M_0—in 2000 and 2010, based on the two-stage identification of the poor proposed by Permanyer (2019). The results are similar to those obtained by the counting method. Differences between the classes of municipalities and between 2000 and 2010 are larger in the case of the incidence of multidimensional poverty (H) than in the case of intensity of poverty (A). Poverty, measured by both H and M_0, remains considerably lower in predominantly urban municipalities and relatively higher in remote municipalities than in intermediate/predominantly rural and remote municipalities.

Multidimensional poverty reduced remarkably between 2000 to 2010 in all classes of municipalities, especially in remote municipalities (a drop of 25 percentage points in the

[22] Lasso de La Vega (2010) presents this first deprivation curve as an increasing step function that is right-continuous, with the horizontal axis displaying the identification cut-offs ranked in decreasing order. In this paper, because there are five classes of municipalities, the graphs are presented in the usual way to facilitate visualization.

[23] When the curves intersect, it is still possible to establish dominance conditions by restricting the set of k.

[24] Taking into account that there are five classes of municipalities and twelve poverty cut-offs, the table of results from the statistical tests is not presented here. However, it is available upon request to the authors.

proportion of poor people). Nonetheless, more than half of Brazil's poor population lived in intermediate and rural municipalities in 2010 (14% and 43% respectively).

Figure 8 presents the distribution of multidimensional poverty across the Brazilian municipalities in 2000 and 2010. The spatial distribution of the multidimensional headcount ratio is similar to that of income poverty, featuring the well-known socioeconomic inequalities between North-Northeast and Centre-South regions. Even though the incidence of poverty is still higher in the North and North–East regions, these regions presented the greatest absolute changes in the incidence of poverty, suggesting the reduction of regional inequalities in Brazil (Table 7).

4.4 Comparison of Approaches

Figure 9 compares the multidimensional poverty reduction in Brazil between 2000 and 2010, through all measures (H, A, and M_0), using the counting identification method (AF) and the two-stage identification (Permanyer 2019). In the AF method, we present the results for all possible poverty cut-offs (k).[25]

The estimates of multidimensional poverty headcount (H) using the two-stage identification method identify 12.7% of the population as multidimensionally poor in 2010, while the AF estimates range between 0.1% (intersection approach)[26] and 57.8% (union approach). The differences between the estimates depend on both the poverty identification function (AF counting versus two-stage in our case) and the corresponding values of k in the counting method. For instance, with full deprivation in education ($k = 0.50$), the incidence of poverty would be 9.9% according to the AF counting approach; and with deprivation in at least three out of six indicators in the standard of living ($k = 0.25$), the incidence of poverty would be 35.4%. The headcount in both methods is approximately the same for $k = 0.417$.

The estimates for the intensity of poverty (A) also show that poverty decreased over time, but less markedly than the dynamic observed for H. According to the two-stage identification method, on average, multidimensionally poor people in Brazil were deprived in 57.4% of the weighted indicators in 2000, and 53.5% in 2010 (Fig. 9). This is an indication of a less pronounced decrease in the proportion of people with the highest numbers of multiple deprivations. Unlike the incidence of poverty, lower values of k in the AF method produce lower estimates of poverty intensity compared to the two-stage approach, because lower k means that more people with fewer deprivations are deemed poor.

The adjusted headcount ratio ($M_0 = H \times A$) combines the incidence and the intensity of poverty. The estimates for the two-stage identification method show that the poor population in Brazil experienced 13.0% of the total possible deprivations they could experience in 2000, and 6.8% in 2010.[27] The estimates for the AF method follow the same trend of the

[25] Table 9 in the Appendix shows all these results.

[26] Based on the microdata sample of the Demographic Census, 0.1% of the Brazilian population corresponded to 218.491 people deprived in all indicators in 2010, of which 60% lived in predominantly rural municipalities.

[27] These results are not comparable with the global MPI since the components of the indices are not the same. Despite the relevance of the global MPI in cross-country comparisons, it is important to remember that the deprivation cut-offs are not suitable for Brazil, particularly with regard to the legislation on education (Brasil 2016). The latest MPI figures (UNDP & OPHI 2020) point out that the Brazilian population experienced 1.6% of the total possible deprivations they could experience in 2015, considering the missing indicator on nutrition and incomplete indicator on child mortality (the survey—PNAD 2015—did not collect the date of child deaths). The multidimensional poverty headcount (H) (i.e. population with a deprivation score of at least 33%) was estimated at 3.8% (7.9 million people in 2015), with an intensity of deprivation (A) of 42.5%. For more information on PNAD 2015, see: https://www.ibge.gov.br/estatistic

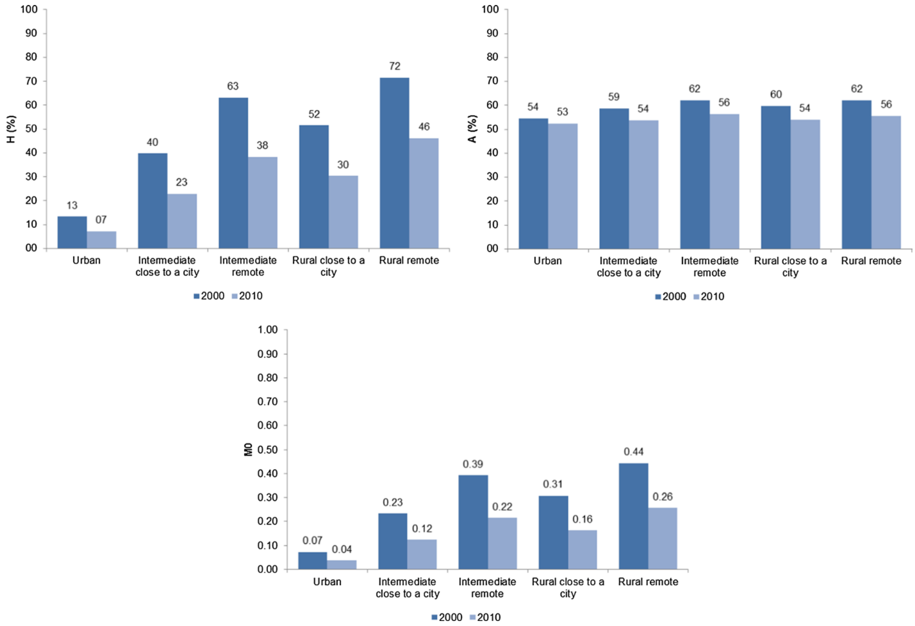

Fig. 7 *H*, *A* and M_0: Permanyer method, by class of municipality—Brazil—2000 and 2010. *Data source*: Demographic Census (IBGE 2017a)

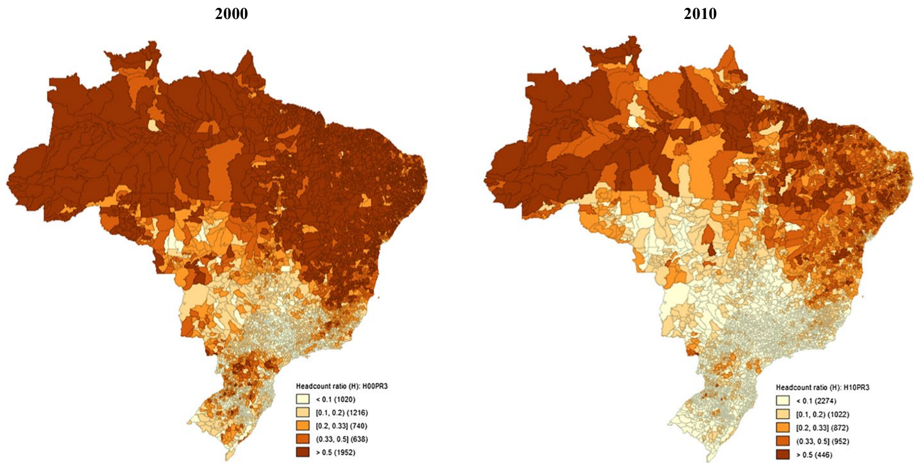

Fig. 8 Multidimensional poverty maps ($H_{\text{Permanyer}}$), by municipality—Brazil—2000 and 2010. *Data source*: Demographic Census (IBGE 2017a)

incidence of poverty. Notwithstanding the significant reduction in the incidence of poverty, the wide disparities between predominantly urban municipalities and intermediate/predominantly rural municipalities still need to be addressed, above all in sanitation and schooling

Footnote 27 (continued)
as/multidominio/condicoes-de-vida-desigualdade-e-pobreza/9127-pesquisa-nacional-por-amostra-de-domicilios.html.

Table 7 Incidence of multidimensional poverty (H) across regions—Brazil—2000 and 2010

Region	Population share (%)		Incidence of poverty (%)		Absolute change	Relative change
	2000	2010	2000	2010	(p.p.)	(%)
North	7.6	8.3	47.3	27.1	– 20.2	– 42.6
North–East	28.2	27.9	45.1	25.5	– 19.6	– 43.4
Centre-West	6.8	7.4	15.1	7.1	– 8.0	– 53.1
South	14.8	14.4	10.0	4.8	– 5.2	– 52.2
South–East	42.7	42.1	8.9	4.9	– 4.0	– 44.7

Data source: Demographic Census (IBGE 2017a)

All changes are statistically significant at $\alpha = 1\%$. Due to rounding, there may be some minor discrepancies in reported numbers

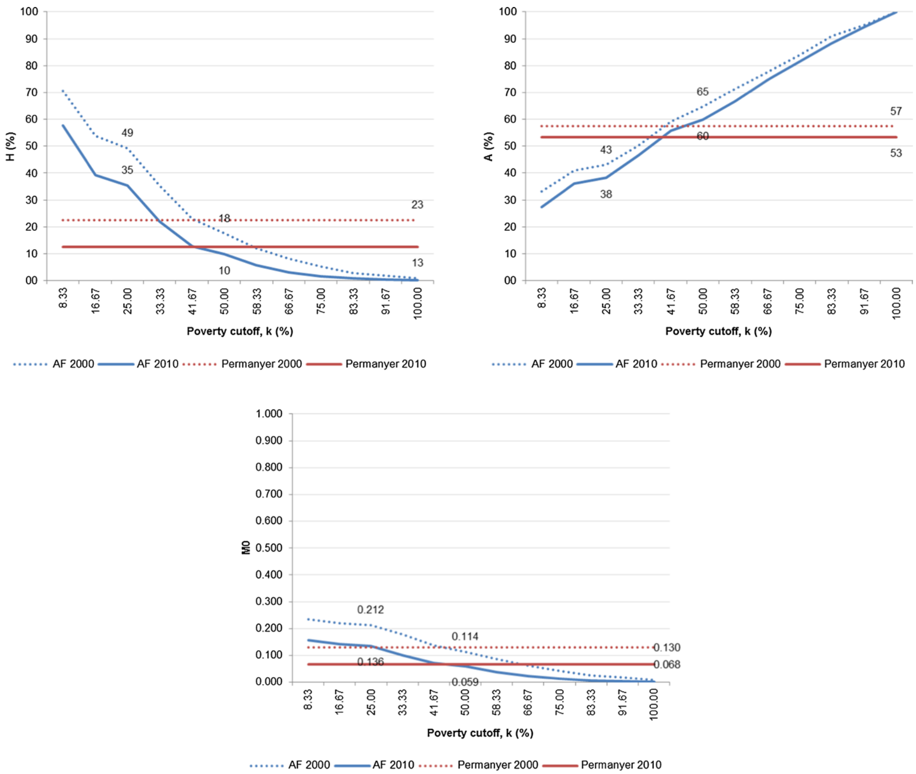

Fig. 9 H, A and M_0: Alkire-Foster and Permanyer methods—Brazil—2000 and 2010. *Data source*: Demographic Census (IBGE 2017a)

achievement. Moreover, the intensity of poverty remains high; that is, the poor continue to face deprivation in basic education and living conditions.

Table 8 Income poverty and multidimensional poverty headcount ratio, by class of municipality—Brazil—2000 and 2010. *Data source*: Demographic Census (IBGE 2017a)

Rural–urban typology	Incidence of poverty			
	2000 (%)	2010 (%)	Absolute change (p.p.)	Relative change (%)
Income poverty				
Brazil	29.6	17.8	− 11.8	− 39.9
Predominantly urban	22.0	12.8	− 9.2	− 41.9
Intermediate close to a city	45.4	27.1	− 18.3	− 40.2
Predominantly rural close to a city	53.5	34.2	− 19.3	− 36.1
Intermediate remote	55.7	39.3	− 16.4	− 29.5
Predominantly rural remote	62.6	47.0	− 15.7	− 25.0
Multidimensional poverty				
Brazil	22.6	12.7	− 9.9	− 44.0
Predominantly urban	13.3	7.2	− 6.1	− 46.0
Intermediate close to a city	39.8	22.9	− 16.8	− 42.4
Predominantly rural close to a city	51.5	30.3	− 21.2	− 41.2
Intermediate remote	63.2	38.3	− 25.0	− 39.5
Predominantly rural remote	71.5	46.2	− 25.3	− 35.4

All changes are statistically significant at $\alpha = 1\%$. Income poverty is defined as monthly household income *per capita* up to R$ 140 in 2010 values (poverty line of the *Bolsa Família Program* in July 2010). Multidimensional poverty is based on the two-stage identification of the poor proposed by Permanyer (2019). Due to rounding, there may be some minor discrepancies in reported numbers

4.5 Intersections between Monetary and Non-Monetary Poverty

This section compares the income poverty reduction in Brazil between 2000 and 2010 to changes in multidimensional poverty. We use the simplest and most used measure—the headcount ratio (H)—and the two-stage identification method for multidimensional poverty measurement.[28] Both monetary and non-monetary approaches are discussed in two complementary ways: marginal distributions and joint distributions of deprivations.

Table 8 shows the dynamics of income poverty and multidimensional poverty. Both measures present similar trends between 2000 and 2010: the income headcount ratio dropped from 30 to 18% (–11.8 p.p. or –40%), and the proportion under multidimensional poverty—in the living standards and education dimensions—dropped from 23 to 13% (–9.9 p.p. or –44%). However, there are some critical differences between the types of municipalities: income poverty fell more markedly (in absolute terms) in municipalities close to a city, while multidimensional poverty fell more remarkably in remote municipalities. In those municipalities with the lowest headcount ratios—predominantly urban and intermediate municipalities close to a city—the absolute rate of change was higher in income poverty: –9.2 and –18.3 p.p., respectively (compared with –6.1 and –16.8 p.p.

[28] Besides, it is arguably harder to compare other monetary and non-monetary indices meaningfully (think, for instance, of trying to compare the monetary square poverty gap against the non-monetary adjusted headcount ratio).

for multidimensional poverty). In the poorest classes of municipalities—rural close to a city, intermediate remote, and rural remote municipalities –, the multidimensional poverty reduction was higher in both absolute and relative terms: –21.2, –25.0, and –25.3 p.p., respectively (compared with –19.3, –16.4 and –15.7 p.p. for income poverty). The poverty reduction was statistically significant at 1% for all classes of municipalities.

In 2010, the incidence of income poverty remained higher than multidimensional poverty in predominantly urban municipalities, and also in intermediate and rural municipalities close to a city. This situation contrasts with that in intermediate and rural remote municipalities. In these areas, multidimensional poverty was substantially higher than income poverty in 2000. However, due to the remarkable fall over the period, the incidence of multidimensional poverty was not significantly different from income poverty in 2010.

Figure 10 illustrates the joint distribution of the Brazilian population in monetary and non-monetary dimensions of poverty. First, there is a clear mismatch between income poverty and multidimensional poverty. Among the population identified as poor based on either poverty perspective (25% in 2010), only one quarter of them (6% in 2010) is simultaneously income poor and multidimensionally poor. The mismatch between monetary and non-monetary poverty may reflect: *i*) the low schooling achievement of the oldest generations; and *ii*) social policies targeted to the poorest (for example, cash transfers, such as *Benefício de Prestação Continuada*[29] and *Bolsa Família*).[30] In other words, these social policies may have alleviated income insufficiency, but may have not impacted significantly on other poverty dimensions.

Second, the intersection between the monetary and non-monetary poor populations is higher in rural and remote areas. For example, the percentage of the population in 2010 experiencing monetary and non-monetary deprivations together ranged from 2% in urban municipalities (14% of the urban poor population and 17% of the total urban population) to 30% in rural remote municipalities (48% of the rural remote poor population and 63% of the total rural remote population). These results may reflect some of the historical disparities throughout the Brazilian territory. People living in rural areas face more pronounced restrictions on access to education and basic sanitation, compared to urban areas. In addition, the longer the distance to urban centres, the greater the difficulty in accessing markets and jobs, limiting income-generating opportunities (Sakamoto et al. 2016).

Finally, Fig. 11 illustrates the extent of the progress made in poverty reduction in Brazil from 2000 to 2010 for three dimensions: standard of living, education, and income. The number of people living in monetary poverty or non-monetary poverty declined by 16 million. A remarkable achievement, given that the population increased by more than 20 million people over this period.

Despite the improvements in all dimensions examined in this study, the lack of income continues to be the main deprivation in the country. The standard of living, which was the second-largest deprivation in 2000, became the lowest one in 2010. All indicators of the standard of living improved in the period, especially electricity, durable consumer goods,

[29] See footnote 2.

[30] Osorio et al. (2011) show that the main changes in income poverty between 2004 and 2009 were a result of inclusive growth through the labour market, real increases in the minimum wage, and increases in coverage and benefits of targeted cash transfers. Regarding non-contributory benefits like BPC and *Bolsa Família*, Barros et al. (2010) found that each of them explains about 10% of the overall reduction in income inequality between 2001 and 2007. Over the period 2001–2011, Hoffmann (2013) estimates the contribution of these benefits to the decline in income inequality between 15 and 20%.

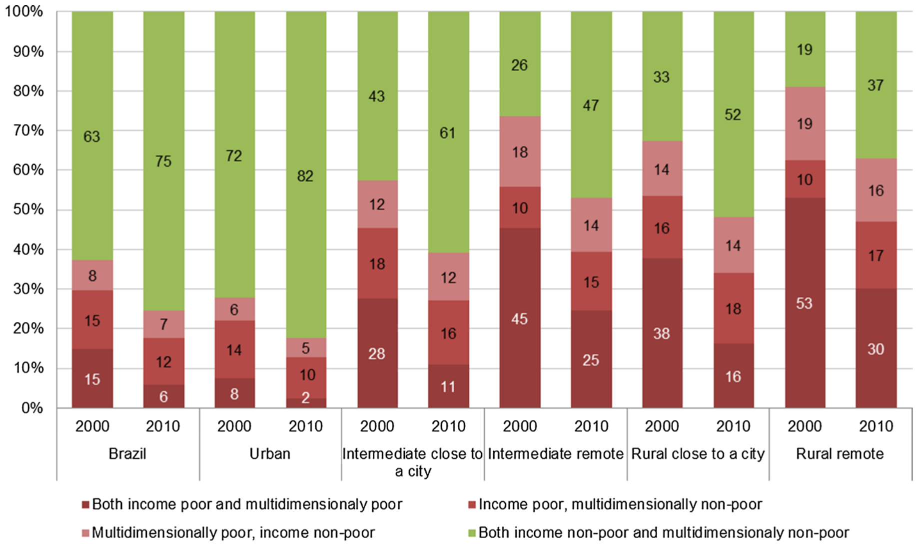

Fig. 10 Joint distribution—income poverty and multidimensional poverty, by class of municipality—Brazil—2000 and 2010. *Note:* Income poverty is defined as monthly household income *per capita* up to R$ 140 in 2010 values (poverty line of the *Bolsa Família Program* in July 2010). Multidimensional poverty is based on the two-stage identification of the poor proposed by Permanyer (2019). *Data source*: Demographic Census (IBGE 2017a)

and private bathrooms in the household. On the other hand, the education dimension has advanced slowly, with 15.9 million people fully deprived in 2000 and 13.8 million in 2010. This result reflects the weight of illiteracy and low schooling achievement among people aged 18 years and above, which corresponded to 70% of the total population in 2010 (IBGE 2017a).

With regard to the overlap between deprivations, the largest interdependence occurs between income and standard of living (21.8 million people deprived in 2000, and 8.6 million in 2010), while only a small share of the population is simultaneously deprived in all dimensions (4.8 million people in 2000, and 1.6 million in 2010). In 2010, among the nearly 47 million people identified as deprived in at least one of three dimensions, 35 million (74%) were deprived in only one of them. Even though most people with some deprivation are deprived only in the income dimension, it is clear from Fig. 11 that complementary policies are necessary to deal with deprivations in the non-monetary dimensions.

The significant reduction in the overlap between deprivations in the standard of living and income highlights the importance of targeted social policies. In addition to economic growth, public policies such as *Bolsa Família* and *Luz para Todos* were vital to alleviate poverty, especially in rural and remote areas, where the incidence of deprivation is relatively larger in all dimensions. The increase in income, combined with access to electricity, allowed the use of many durable goods, such as refrigerators and other household appliances. Electricity also allows improving sanitation conditions, for instance, by having a well pump and piped water at home. All of these factors contribute to well-being, improving the health conditions and time use of household members, particularly women.

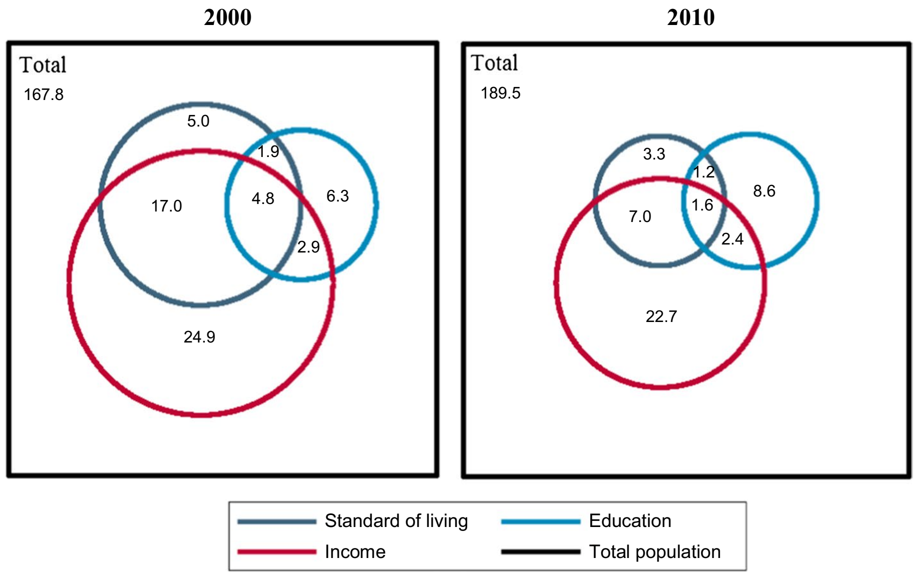

Fig. 11 Deprivations in multiple dimensions (millions of people) —Brazil—2000 and 2010. *Data source*: Demographic Census (IBGE 2017a)

5 Concluding Remarks

In this paper, we analysed multidimensional poverty in Brazil using the largest source of information for socioeconomic analysis available in the country: the microdata of the Demographic Censuses in 2000 and 2010. The paper brings in some main contributions to the literature on multidimensional poverty. The first one is to propose multidimensional poverty measures that take more accurately into account the diverse array of deprivations in Brazil. We compared two methods—the counting identification of the poor of the AF method and the two-stage identification model proposed by Permanyer—and the intersections with the estimates obtained using monetary poverty. The second contribution is to present a broad analysis of multidimensional poverty in Brazil in a period of significant socioeconomic changes, when poverty, both monetary and multidimensional, fell significantly. The decomposition of the multidimensional poverty by classes of rural/urban municipalities also highlighted huge differences in the level and dynamics of poverty in Brazil.

Our results highlight that, independently of the poverty cut-off, both the multidimensional headcount ratio (H) and the adjusted headcount ratio (M_0) are higher in remote municipalities than in municipalities close to a city, whether they are classified as intermediate or rural municipalities. In other words, the distance to an urban centre seems to be more associated with the incidence and intensity of poverty than population size or density. These results reinforce the importance of distance in the process of rural development. The proximity to an urban centre matters for both better opportunities of employment (and income) in the service and manufacturing sectors, and better access to basic requirements for adequate living conditions and educational achievement. In fact, the differences between remote localities and localities close to a city are higher regarding multidimensional poverty than income poverty.

Nonetheless, poverty declined most sharply in less developed areas—rural and intermediate municipalities, as in the North and North-East regions, contributing to reduce regional as well as urban–rural inequalities related to the multiple dimensions of poverty. More importantly, poverty reduction in less populated and remote areas was larger in non-monetary dimensions—the standard of living and education—compared to the income perspective. Public policies targeted to the poorest localities, such as the rural electrification program (*Luz para Todos*), may have played a major role in alleviating basic infrastructure needs. In turn, income poverty decreased faster in rural/intermediate municipalities close to a city, probably hinting at the role of market access in rural economic development. Off-farm jobs have played a significant role in improving income in rural Brazil, and the proximity to the city decidedly increases the odds of a rural resident being employed in the urban economy.

The comparison between two different methods of identification of the multidimensionally poor also provided useful elements to understand the implications of the methodological approach to public policies. The AF method has been widely used. It stands out for its simplicity, ease of understanding, and because the resulting measures of the counting approach satisfy a set of desirable properties in poverty measurement. However, the arbitrariness usually involved in the weighting distribution among the indicators has been one of the method's main criticisms, besides the possibility of compensation among any pairs of indicators. Depending on the weighting structure and the poverty cut-off, the method may overestimate or underestimate the poor population. Even though different methods may result in the same headcount ratio, different people would be identified as poor, which is a crucial issue in targeting anti-poverty policies. Thus, the two-stage identification of the poor, based on a set of poverty profiles, significantly contributes to the identification step, as it considers the substitutability and complementarity between indicators and dimensions to define the target population. In the empirical application of this method, a person was identified as multidimensionally poor if he/she was deprived in standard of living or fully deprived in education. The lower the poverty cut-off in the AF method, the greater the overestimation of both H and M_0 compared to the estimates from the two-stage identification model.

The intersections between monetary and non-monetary poverty in Brazil also highlighted that only a fraction of the population identified as poor based on either poverty perspective is simultaneously poor in both of them. The diversity and lack of coordination of social policies targeted at the poorest in Brazil may help explain this result. For example, cash transfers may reduce income poverty, but may have little, if any, effect on other dimensions of poverty (education, for instance). The mismatch between income poverty and multidimensional poverty also differs considerably between classes of municipality, with the greater overlaps in the remote municipalities. Huge regional heterogeneities in Brazil may largely explain these results. The income poor living in remote and rural areas may face greater restrictions on access to basic infrastructure and education than those living in urban areas. In turn, some basic achievements, such as access to electricity and literacy, may be almost universal in the most developed urban centres, even among the income poor. Despite the improvements in all dimensions over time, the incidence of income poverty remains the main problem among the analysed dimensions. Great progress was made in the standard of living, especially in the access to electricity, durable consumer goods and private bathrooms in the households, while the education dimension has advanced slowly. The school attendance of the 15–17 age group and the schooling achievement among the adult population are some of the central issues to the country's development. Ending poverty in all its forms is still a major challenge. A considerable proportion of the population is deprived in basic indicators of the standard of living and education, with the worse conditions in the remote municipalities.

Multidimensional poverty measurement is a first and essential step towards providing constitutional rights and opportunities for those most deprived. The multidimensional poverty index presented in this paper aims to contribute towards proposing measures that may promote better living standards for those people facing the greatest deprivations in Brazil. Although limited to only two dimensions and lacking relevant indicators of well-being, such as child nutrition, the basic achievements in the standard of living and education alone would be extraordinary progress. Deprivation in living conditions and basic education limits the opportunities for people's achievements, and these opportunities depend on location. In sum, poverty alleviation in Brazil requires facing the deprivation of opportunities so that everyone can live a decent life in society, regardless of where they live.

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Availability of Data and Materials Available upon request to the authors.

Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no conflict of interest.

Appendix

See Appendix Table 9.

Table 9 H, A and M_0: Alkire-Foster counting method and two-stage identification of the poor—Brazil—2000 and 2010

Method	k (%)	H (%)		A (%)		M^0	
		2000	2010	2000	2010	2000	2010
Alkire–Foster	8.33	70.5	57.8	33.2	27.3	0.234	0.157
	16.67	53.8	39.3	40.9	36.1	0.220	0.142
	25.00	49.3	35.4	43.1	38.3	0.212	0.136
	33.33	35.4	21.9	50.2	46.4	0.178	0.102
	41.67	23.1	12.8	59.2	55.8	0.137	0.071
	50.00	17.6	9.9	64.7	59.9	0.114	0.059
	58.33	12.1	5.8	71.4	66.8	0.086	0.039
	66.67	8.1	3.0	77.7	74.6	0.063	0.023
	75.00	5.2	1.6	84.0	81.4	0.044	0.013
	83.33	2.9	0.8	90.9	88.4	0.027	0.007
	91.67	1.9	0.4	95.0	94.3	0.018	0.003
	100.00	0.8	0.1	100.0	100.0	0.008	0.001
Two-stage identification		22.6	12.7	57.4	53.5	0.130	0.068

Data source: Demographic Census (IBGE 2017a)

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